Commonality Specifications, Merged Models, and Partial Morphisms

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Notation and Prerequisites

- Let $M$ be a (e.g. class or data) model. $A$ is a snapshot of data typed over $M$, i.e. an assignment $\tau : A \rightarrow M$.
- For each $M$ let’s denote with $\mathcal{M}$ the model space of these snapshots.
- Given a collection of (locally) consistent data snapshots $A_1, \ldots, A_n$ from different models spaces $\mathcal{M}_1, \ldots, \mathcal{M}_n$ and ...
- ...inter-model constraints that spread over some of the $M_i$
- Goal: Formally / virtually understand the $n$ related spaces as one comprehensive collage ...
- ...to apply known methods for consistency checking and restoration.
- Assumption: All artefacts are basically graph-based.
**Intermodel Constraint**: If there is already a bed assigned to the patient, then there should be an appointment scheduled whenever there is a severe cholesterol test.
Data Commonalities

We can not check consistency, if we don’t know whether patient Mary in $A_1$ (typed over $M_1$) is the same as Marie in $A_2$ (typed over $M_2$):

<table>
<thead>
<tr>
<th>Samenesses:</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>Mary</td>
<td>John</td>
<td>Bob</td>
</tr>
<tr>
<td>$A_2$</td>
<td>Marie</td>
<td>John</td>
<td>NULL</td>
</tr>
<tr>
<td>$A_3$</td>
<td>NULL</td>
<td>John</td>
<td>Bob</td>
</tr>
</tbody>
</table>

Coded as ternary span requires the use of *partial* mappings:

![Diagram of partial mappings]
Partial Morphisms are Spans of Total Morphisms

\[ A_0 \]

\[ A = \quad \text{dom}(a_1) \ldots \text{dom}(a_n) \]

\[ a_1 \quad \ldots \quad a_n \]

\[ A_1 \quad \ldots \quad A_n \]

\[ M_0 \]

\[ M = \quad \text{dom}(m_1) \ldots \text{dom}(m_n) \]

\[ m_1 \quad \ldots \quad m_n \]

\[ M_1 \quad \ldots \quad M_n \]
Let \( \text{merge} \) be the operation that takes a (graph) diagram and computes its colimit w.r.t. to total morphisms.

\[
\begin{array}{ccc}
\mathcal{A} & \longrightarrow & \text{merge}(\mathcal{A}) \\
\downarrow \tau & & \downarrow \tau \\
\mathcal{M} & \longrightarrow & \text{merge}(\mathcal{M})
\end{array}
\]

Multi-Model: Given data \( \mathcal{A} \) and inter-model-constraints \( C \) on \( \mathcal{M} \).

\[
\mathcal{A} \models C?
\]

Reduced to: Given data \( \mathcal{A}^+ := \text{merge}(\mathcal{A}) \) and inter-model-constraints \( C \) imposed on \( \mathcal{M}^+ := \text{merge}(\mathcal{M}) \).

\[
\mathcal{A}^+ \models C?
\]

see P. Stünkel et al: *Multimodel Correspondence through Inter-Model Constraints* @ BX’18
Update Propagation and Consistency Restoration

Colimit $F$ of $\mathcal{M}$ (the federated system):

Inter-Model Constraint is imposed on $F$ (yielding $F'$). Local update $A_i \rightarrow A'_i$ is put into data typed over $F'$ and then projected back (get) to $A_j$ ($j \neq i$) and possibly to $A_i$ (amendments!), yielding a ternary delta-lens, which can be ”implemented” as a ternary span of asymmetric delta-lenses.