

# Regarding the Optimality of Speedup Bounds of Mixed-Criticality Schedulability Tests

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## 1 Introduction

Much existing research on Mixed-Criticality (MC) scheduling (see [7] for a review) has focused on dealing with the Vestal model [15], where different WCET estimations of a single piece of code are provided. This is typically a consequence of different tools for determining worst case execution time (WCET) bounds being more or less conservative than each other. It is known [1] that mixed criticality (MC) scheduling under such model is highly intractable, such that polynomial-time optimal solution is impossible unless  $P = NP$ . As a result, *speedup bound* is widely used in MC scheduling for measuring how close to optimal is a given schedulability analysis.

- A schedulability test has *speedup factor* of  $s (s \geq 1)$ , if any task set that is schedulable by any algorithm on a given platform with processing speed of 1, it will be deemed schedulable by this test upon a platform that is  $s$  times as fast.

Of course when deriving MC schedulers and associated schedulability tests, one of the goals is to identify/prove a relative small speedup bound (that is closer to 1). A minimum possible speedup is often presented as the “*optimal* speedup bound” of a given MC scheduling problem. However, we would like to point out that:

- Optimality of scheduler should not be derived against optimal speedup bounds.

## 2 Non-Optimal Schedulers with Optimal Speedup Bounds

For scheduling (dual-criticality) Vestal job set on a uniprocessor platform, it has been shown [2] that OCBP algorithm (following the idea of Audsley’s priority assignment mechanism) has an optimal speedup bound of  $(\sqrt{5} - 1)/2$ . However, several algorithms has been identified to *strictly dominate* OCBP; e.g., Lazy Priority Adjustment [10], LE-EDF [12] [11] — they have the same speedup, yet the latter has better schedulability at *all* time. Similar results can be observed when we consider the scheduling of Vestal task as well. It has been shown that  $4/3$  is the best speedup that any non-clairvoyant scheduler can achieve. Upon proposing a speedup-optimal uniprocessor scheduler named EDF-VD [3], improvements on the schedulability can still be made,

e.g., [9] [8]. As for the multiprocessor case, it is proved [4] that both MC-Fluid [13] and MCF [4] achieve the optimal speedup of  $4/3$ . However, MCF is a simplified version of (and is dominated by) MC-Fluid. Moreover, improvements on schedulability can be further made to MC-Fluid [14].

### 3 Speedup over Non-Clairvoyance?

When deriving speedup bounds, in most of the existing works of the community, the proposed algorithm is compared with a *clairvoyant optimal scheduler*, and adapts the necessary conditions for MC schedulability. This may not be a very fair way of comparison, since the penalty for unawareness of the future is applied into the speedup bounds. Following the varying-speed MC model [6] [5], we have identified an on-line optimal<sup>1</sup> scheduler in [11] that has a speedup factor significantly greater than 1 when comparing to an optimal clairvoyant algorithm. However, such a speedup factor only reflects the price one must pay for not knowing the future (or the difficulty of the scheduling problem itself) — it has nothing to do with the MC scheduler design any more.

Since MC schedulability analysis is for off-line verification of correctnesses of real-time systems, all possible scenarios should be taken into consideration (which is non-clairvoyance). We believe speedup results comparing to optimal non-clairvoyance schedule may be worth investigating for MC systems.

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<sup>1</sup>By on-line optimal, if our algorithm returns unschedulable for an MC instance, then no algorithm can guarantee correctness without making lucky guesses to the future.