Open Problems in Supervisory Control of Partially-Observed Discrete Event Systems

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Supervisory Control under Partial Observation

Control Engineering Perspective

\[ S_P : E^* \rightarrow \Gamma \]

\[ S_P : P(\mathcal{L}(G)) \rightarrow \Gamma \text{ where } \Gamma := \{ \gamma \in 2^E : E_{uc} \subseteq \gamma \}. \]

Closed-loop Behavior: \( \mathcal{L}(S_P/G) \) and \( \mathcal{L}_m(S_P/G) \)
Discrete Event Systems: Logical Properties

**Safety:**
- no illegal *states* reached
- no illegal *substrings* executed
- Formally: Specification automaton $H$

\[
L_a := L(H) \subseteq L(G)
\]
\[
L_{am} := L(H) \cap L_m(G) \subseteq L_m(G)
\]
- Can be mapped to state specification
Nonblocking: no deadlocks or livelocks
Comments on Supervisor:

- The system has its uncontrolled behavior (there is a “plant”).

- The supervisor disables events to restrict the system behavior; i.e., the supervisor enables a set of events.

- We want the supervisor to be maximally permissive.
Comments on Supervisor:

- The system has its uncontrolled behavior (there is a “plant”).
- The supervisor disables events to restrict the system behavior; i.e., the supervisor enables a set of events.
- We want the supervisor to be maximally permissive.
- Optimality criterion is set inclusion
- Only disable an event if absolutely necessary to guarantee safety and nonblocking
Existence Conditions

Theorem (Controllability and Observability Theorem)

Given a specification language $K$, there exists a non-blocking supervisor $S_P$ such that $L_m(S_P/G) = K$ if and only if

C1. $\overline{K}E_{uc} \cap L(G) \subseteq \overline{K}$ (Controllability)

C2. $\forall s\sigma \in \overline{K}, \sigma \in E_c : [P^{-1}P(s)]\sigma \cap L(G) \subseteq \overline{K}$ (Observability)

C3. $\overline{K} \cap L_m(G) = K$ ($L_m(G)$-closure)
Problem (Basic Supervisory Control and Observation Problem: Non-Blocking and Maximal (BSCOP-NB\textsuperscript{max}))

Let $G$ be the plant and $L_{am}$ be the non-prefix-closed specification language. Find a supervisor $S_P : E^*_o \rightarrow \Gamma$ such that

\begin{enumerate}
\item[C1.] $\mathcal{L}(S_P/G) \subseteq \overline{L_{am}}$ \hspace{1cm} (Safety)
\item[C2.] $S_P/G$ is non-blocking \hspace{1cm} (Non-blockingness)
\item[C3.] For any $S'_P$ satisfying C1 and C2, we have

\[ \mathcal{L}(S_P/G) \not\subseteq \mathcal{L}(S'_P/G) \] \hspace{1cm} (Maximal Permissiveness)
\end{enumerate}
Comments on BSCOP-NB$^{max}$:

Controllability and $L_m(G)$-closure are preserved under union.

Observability is not preserved under union.

The solutions are locally maximal.

Sub-optimal solutions:

- Stronger properties: normality and relative observability 
  \[\text{[Lin and Wonham, 1988, Cieslak et al., 1988, Cai et al., 2015]}\]

- State-based property: \[\text{[Takai and Ushio, 2003]}\]

- When non-blockingness is relaxed, solution is found by \[\text{[Ben Hadj-Alouane et al., 1996, Yin and Lafortune, 2014a]}\].

- When maximality is relaxed, solution is found by \[\text{[Inan, 1994, Yoo and Lafortune, 2006]}\].

BSCOP-NB$^{max}$ has been solved in our recent work \[\text{[Yin and Lafortune, 2014b]}\].

Related Works in Different Frameworks:

\[\text{[Arnold et al., 2003, Pinchinat and Riedweg, 2005, Chatterjee et al., 2006, Thistle and Lamouchi, 2009]}\]
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- Controllability and $\mathcal{L}_m(G')$-closure are preserved under union.
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  - The solutions are \textit{locally maximal}
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    - Stronger properties: normality and relative observability
      \cite{Lin1988,Cieslak1988,Cai2015}
    - State-based property: \cite{Takai2003}
- When non-blockingness is relaxed, solution is found by \cite{BenHadj-Alouane1996,Yin2014a}.
- When maximality is relaxed, solution is found by \cite{Inan1994,Yoo2006}.
- BSCOP-NB$^{max}$ has been solved in our recent work \cite{Yin2014b}.
- Related Works in Different Frameworks: \cite{Arnold2003,Pinchinat2005,Chatterjee2006,Thistle2009}
Non-Uniqueness of Locally Maximal Solutions

$E_c = \{c\}, E_o = \{o\}$
Non-Uniqueness of Locally Maximal Solutions

Two incomparable solutions

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Non-Uniqueness of Locally Maximal Solutions

\[ E_c = \{c\}, E_o = \{o\} \]

Two incomparable solutions

How to contain a desired behavior?
Problem (Range Control Problem for Safety and Non-blockingness)

Let $G$ be the plant and $L_r$ and $L_{am}$ be two non-prefix-closed languages. Find a supervisor $S_P : E_o^* \rightarrow \Gamma$ such that

C1. $L_r \subseteq \mathcal{L}(S_P/G) \subseteq \overline{L_{am}}$

C2. $S_P/G$ is non-blocking

C3. For any $S'_P$ satisfying C1 and C2, we have

$\mathcal{L}(S_P/G) \not\subset \mathcal{L}(S'_P/G)$
Comments on Range Control Problem: Open Problems

Comments on Range Control Problem:

The range control problem is still open. Even the following simplified problems are open:

Non-blocking Range Control Problem (No Maximality):
if we only require

\[ C_1. \text{L}^r \subseteq \text{L}(\text{S}/\text{G}) \subseteq \text{L}_{am} \]

C2. \text{S}/\text{G} is non-blocking

Maximal Safe Range Control Problem (No Non-blockingness):
if we only require

\[ C_1. \text{L}^r \subseteq \text{L}(\text{S}/\text{G}) \subseteq \text{L}_{am} \]

C2. For any \text{S} satisfying C1 and C2, we have

\[ \text{L}(\text{S}/\text{G}) \not\subset \text{L}(\text{S}'/\text{G}) \]
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    if we only require
    \[ C_1. \quad L_r \subseteq \mathcal{L}(S_P/G) \subseteq \overline{L_{am}} \]
    \[ C_2. \quad \text{For any } S'_P \text{ satisfying } C_1 \text{ and } C_2, \text{ we have} \]
    \[ \mathcal{L}(S_P/G) \nsubseteq \mathcal{L}(S'_P/G') \]
Comments on Range Control Problem: Difficulties

- Enabling less now may result in a larger future behavior

\[ E_c = \{ c \}, E_o = \{ o \} \]
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The verification of maximality is still open
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- Enabling less now may result in a larger future behavior

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- The verification of maximality is still open

- The maximal solutions in current methodologies are only a particular class of maximal solutions, i.e., greedy maximal

[Ben Hadj-Alouane et al., 1996, Yin and Lafortune, 2014a, Yin and Lafortune, 2014b]
Decentralized Supervisory Control

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Fusion

Plant G

$s_1(s)$

$s_i(s)$

$s_n(s)$

$S_1$

$P_1$

$S_2$

$P_2$

$S_n$

$P_n$
Existence Conditions

Theorem (Controllability and Co-Observability Theorem)

Given a specification language $K$, there exist non-blocking supervisors $S_1$ and $S_2$ such that $\mathcal{L}_m(S_1 \land S_2/G) = K$ if and only if

C1. $\overline{KE_{uc}} \cap \mathcal{L}(G) \subseteq \overline{K}$ (Controllability)

C2. $\forall \sigma \in \mathcal{L}(G) \setminus \overline{K}, \sigma \in E_c, \exists i \in I^c(\sigma) : [P_i^{-1} P_i(s)]\sigma \cap \overline{K} = \emptyset,$
where $I^c(\sigma) = \{ i \in \{1, 2\} : \sigma \in E_{c,i}\}$ (Co-Observability)

C3. $\overline{K} \cap \mathcal{L}_m(G) = K$ ($\mathcal{L}_m(G)$-closure)

Conjunctive architecture (other architectures exist with associated versions of co-observability)

S. Lafortune (UMich)

Supervisory Control of DES

02-06 February 2015

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C2. $\forall s\sigma \in L(G) \setminus \overline{K}, \sigma \in E_c, \exists i \in I^c(\sigma) : [P_i^{-1}P_i(s)]\sigma \cap \overline{K} = \emptyset$, where $I^c(\sigma) = \{i \in \{1, 2\} : \sigma \in E_{c,i}\}$ \hspace{1cm} (Co-Observability)

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- Conjunctive architecture (other architectures exist with associated versions of co-observability)
Decentralized Control Problem for Safety and Non-blockingness

Problem (Decentralized Control Problem for Safety and Non-blockingness)

Let $G$ be the plant and $L_{am}$ be the non-prefix-closed specification language. Find supervisors $S_1$ and $S_2$ such that

C1. $\mathcal{L}(S_1 \wedge S_2/G) \subseteq \overline{L_{am}}$

C2. $S_1 \wedge S_2/G$ is non-blocking

Comments: This problem is undecidable \cite{Tripakis, 2004, Thistle, 2005}.

Therefore, the decentralized maximal control problem, range control problem, and maximal range control problem are all undecidable for non-blocking specification $S$. Lafortune (UMich)
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Problem (Maximal Decentralized Control Problem for Safety)

Let $G$ be the plant and $L_a$ be the prefix-closed specification language. Find supervisors $S_1$ and $S_2$ such that

**C1.** $\mathcal{L}(S_1 \land S_2/G) \subseteq L_a$

**C2.** For any $S_1', S_2'$ satisfying above, $\mathcal{L}(S_1 \land S_2/G) \not\subseteq \mathcal{L}(S_1' \land S_2'/G)$

Comments:

This problem is still open.

Lack of "Information State". Control decisions are mutually dependent.

The solutions are characterized as Nash equilibria by [Overkamp and van Schuppen, 2000].

The problem is solvable if the second requirement is relaxed.

Solution is $\{\epsilon\} \downarrow C$: disable all controllable events.

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<table>
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<tr>
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<th>Safe</th>
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<th>Safe+NB</th>
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<tr>
<td>Centralized Upper Bound</td>
<td>[1,2,3]</td>
<td>[4]</td>
<td>[5]</td>
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<tr>
<td>Centralized Range</td>
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<tr>
<td>Decentralized Range</td>
<td>[2,7]</td>
<td>OPEN</td>
<td>Undecidable</td>
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</tr>
</tbody>
</table>

[1][Lin and Wonham, 1988]  [2][Cieslak et al., 1988]  [3][Rudie and Wonham, 1990]  [4][Ben Hadj-Alouane et al., 1996]  [5][Yoo and Lafortune, 2006]  [6][Yin and Lafortune, 2014b]  [7][Rudie and Wonham, 1992]  [8][Tripakis, 2004]  [9][Thistle, 2005]

Table: Summary of Problems in Partially-Observed DES literature
Deadlock-freeness Relaxation

When Non-Blockingness is relaxed to Deadlock-Freeness

$L$ is deadlock-free if $\forall s \in L, \exists \sigma \in E: s\sigma \in L$

The centralized deadlock-free range control problem is still open.
Therefore, maximal deadlock-free range control problem is also open.
The decentralized deadlock-free control problem is still open.
Therefore, the decentralized maximal, range or maximal range control problems are all open for deadlock-freeness specification.
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- The decentralized deadlock-free control problem is still open. Therefore, the decentralized maximal, range or maximal range control problems are all open for deadlock-freeness specification.
Games for synthesis of controllers with partial observation.

Centralized and distributed algorithms for on-line synthesis of maximal control policies under partial observation.

Relative observability of discrete-event systems and its supremal sublanguages.
In *IEEE Transactions on Automatic Control*.

Algorithms for omega-regular games with imperfect information.

Supervisory control of discrete-event processes with partial observations.

Nondeterministic supervision under partial observations.

On observability of discrete-event systems.
Maximal solutions in decentralized supervisory control.

A decidable class of problems for control under partial observation.

The infimal prefix-closed and observable superlanguage of a given language.

Think globally, act locally: Decentralized supervisory control.

Effective computation of an $L_m(G)$-closed, controllable, and observable sublanguage arising in supervisory control.

Undecidability in decentralized supervision.

Effective control synthesis for partially observed discrete-event systems.
Undecidable problems of decentralized observation and control on regular languages.

A general approach for synthesis of supervisors for partially-observed discrete-event systems.
In *19th IFAC World Congress*, pages 2422–2428.

Synthesis of maximally permissive non-blocking supervisors for partially observed discrete event systems.

Solvability of centralized supervisory control under partial observation.